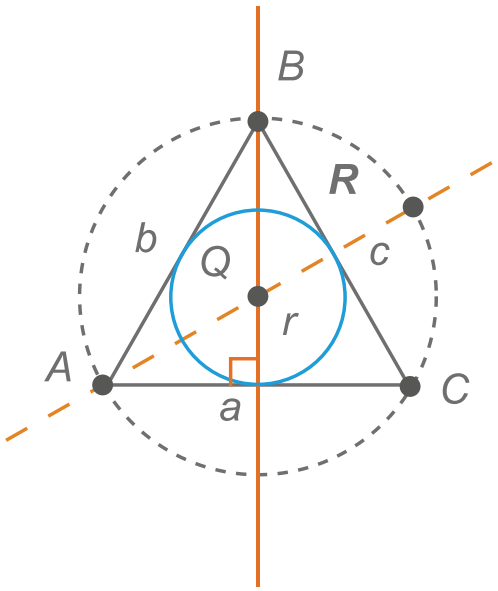

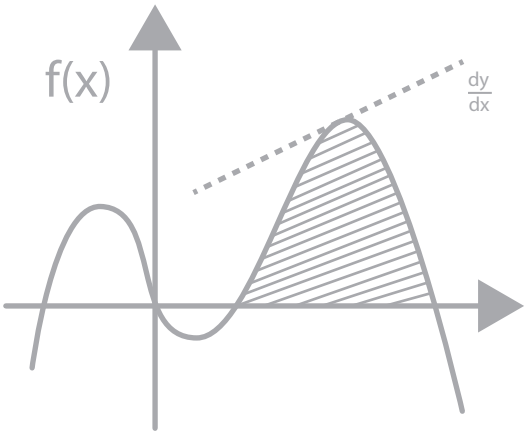

MAT135H

Calculus I

Week 1

Fall 2021

TE明星导师Frank – 阿福老师

在数学的领域中 提出问题的艺术比解答问题更为重要

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MAT135

Calculus 1 Syllabus

University of Toronto **Fall 2021**

This syllabus is for the in-person version of the course. The in-person and the online versions of MAT135 are run completely separately, including assessments and final exam.

Course Objectives

The course has four objectives:

1. **Calculus concepts.** We will study various concepts in calculus (functions, limits, derivatives, integrals, series). We want you to build a mental framework of calculus that serves as a foundation for future learning.
2. **Problem solving.** We want to train you in the art of problem solving. In your future career, you will see problems that you haven't seen before. We want you to be able to figure out by yourself how to adjust the methods you know to fit new situations and to be confident with your solutions. You can only achieve this by understanding rather than memorizing formulas and methods.
3. **Applications.** We will introduce you to many different ways to apply calculus to understand the world around us. We want to make you comfortable to take in a situation and see it through the lens of calculus.
4. **Scientific communication.** We want you to see yourself as a confident and capable user and communicator of mathematics.

Essential Questions

In this course we will address the following questions:

1. Why should we represent a single relationship in different ways?
2. What is infinity? What is an infinitesimal?
3. How do we model the real-world with mathematics?
4. What is speed, and how do you measure it? What are rates, and how do you measure them?
5. How can you solve novel problems that are unlike any you've encountered before?
6. What do good readers and writers of math do?

Course Topics

We will work through the following units in MAT135, corresponding to the textbook sections below.

1. **1.1–1.6.** Modelling with Functions: How do we use mathematics to describe related quantities?
2. **2.1, 1.7–1.9.** Limits: How do we work with the infinitely small and the infinitely large?
3. **2.2–2.6.** The Derivative: In what different ways can rates of change be represented? How are rates of change described and used?
4. **3.1–3.7.** Computing Derivatives: How are derivatives efficiently computed?
5. **3.9, 4.1–4.4, 4.6–4.7** Using the Derivative: How can we use the derivative to solve complex problems from the sciences?
6. **5.1–5.3** The Area Problem: How is the rate of change problem related to the area problem?

Is MAT135H the right calculus course for you?

MAT135H is the first in the sequence of calculus courses for students intending to major in science, and is the prerequisite for MAT136H1. Other calculus courses offered by the Faculty of Arts & Sciences include MAT133Y, MAT137Y, and MAT157Y.

- **MAT133Y** introduces students to both calculus and linear algebra and is intended for Commerce students. It does not cover as much calculus as MAT135H1 and MAT136H1, and is not a valid prerequisite for most math and statistics courses.
- Both **MAT137Y** and **MAT157Y** are proof-based approaches to calculus, intended for students who are planning to take further mathematics courses. These courses go further into the mathematical basis of calculus, whereas the MAT135/MAT136 sequence will focus more on applications.

Course Components

What we assume you know

Historically, students who come into a calculus course with strong knowledge of algebra and functions perform far better than students who have weaker skills.

Further, students who work to improve their pre-calculus and mathematics study skills attain greater mastery of calculus[1,2]. Other recent research shows that a solid precalculus foundation is a very good predictor of success in calculus[3].

To help you prepare for the course, the Math department prepared the website

`http://uoft.me/precalc`

This website contains tutorials, examples, and self-diagnostic quizzes of the topics we expect you to have learned in high school.

How to learn?

Pre-class	Class	Post-class	Tutorial
READ textbook	ATTEND class	REVIEW what you learned	APPLY
ANNOTATE textbook	TEAMWORK	DO WeBWorK	COMMUNICATE
WATCH videos	MAKE connections	ASK questions	
		USE additional resources	
		COMPLETE ACTs	

Pre-class essentials

Before each class, you will spend about 30 minutes learning the elementary concepts of the material that we will work on during class.

We have created short videos to introduce you to the basics of each concept. We expect you to watch them before each class. The videos are linked from the course website.

We will also be using Perusall to establish a community where you can read the textbook, annotate it, ask questions about it, and engage in discussions with your peers. These will be assigned at the end of each week and due before your first lecture of the week.

Classes will be designed towards students who have done the pre-class essentials. If you did not do so, you cannot expect to learn anything in class.

Textbook

The textbook we will be using is ***Calculus: Single Variable* by Deborah Hughes-Hallett, et al, 7th edition**.

To get the book, you need to create an account with Perusall at `https://app.perusall.com` and use the course code: `GALVAO-SOUSA-JFHWM`. The price for the etextbook together with Perusall is: $100 for 50 years!

[1] `http://wacra.org/publicdomain/ijcra%20xxi_iii_pg232-236%20agustin.pdf`

[2] `http://dx.doi.org/10.1080/10511979808965902`

[3] `https://news.harvard.edu/gazette/story/2018/07/masters-of-calculus-come-prepared-harvard-study-shows/`

Class

You will have three weekly hours of class. Classes will be a mix of the instructors introducing concepts and students working with each other. We won't record classes. We will post the questions we use in class, but no solutions or anything else. The point of class is for you to actively engage with the material, not to watch or read somebody else's solutions.

It is mandatory to use a mask covering your nose and mouth during every lecture.

Teaching Team.

Instructors	Email	Section
Dr. Bernardo Galvão-Sousa **(coord.)**	beni@math.toronto.edu	LEC0101
Dr. Tomas Dominguez	tomas.dominguezchiozza@mail.utoronto.ca	LEC0102
Dr. Dmitry Panchenko	panchenk@math.toronto.edu	LEC0201
Dr. Lindsey Shorser	lshorser@math.toronto.edu	LEC0202
Dr. Emile LeBlanc	leblanc@math.toronto.edu	LEC0301+LEC0601
Dr. Assaf Bar-Natan	assaf.bar.natan@mail.utoronto.ca	LEC0302
Dr. Teemu Tyni	teemu.tyni@utoronto.ca	LEC0401
Dr. Daniel Álvarez	daniel.alvarez@utoronto.ca	LEC0402
Dr. Caelan Wang	caelan.wang@utoronto.ca	LEC0501
Dr. Joshua Lackman	jlackman@math.toronto.edu	LEC0502
Dr. Tianyu Zhou	ty.zhou@mail.utoronto.ca	LEC0602

Post-class

WeBWorK assignments are your chance to get a benchmark on how well you know the concepts covered that week. They serve as a good practice for term tests and the final exam. Don't start with WeBWorK. Finish with it. Doing WeBWorK should be the last step in your learning process for each module. Don't jump right to it.

Based on the classroom and tutorial experience, you will now have an idea which concepts are clear to you and which you should work on. There are many resources at your disposal to help you study:

Suggested problems can be found on Quercus

Ed discussion board where you can ask questions to instructors and peers

Drop-in hours where you can come ask questions to instructors or TAs

Math Learning Center (MLC) in [ROOM] where you can work and ask questions to instructors or TAs

Recognized Study Groups (RSG) where you can join/start a study group (`https://uoft.me/RSG`)

Academic Success Centre offers a variety of services and programming to help students meet their academic and personal goals at the University (`https://www.studentlife.utoronto.ca/asc`)

Tutorials

Each week, you will attend one tutorial, a class of about 30 students led by your own Teaching Assistant (TA). The purpose of tutorials is to improve your problem-solving and communication skills, and to provide you opportunities to collaborate with other students.

Check your tutorial assignment. You must attend the tutorial that you are assigned to according to ACORN. Tutorials start on Thursday, September 23.

It is mandatory to use a mask covering your nose and mouth during every lecture.

Head TAs. Matthew Murphy, Petr Smirnov, and Sarah Machado-Marques

TAs. Akira, Alykhan, Amalrose, Ava, Charles, Charlie, Chayim, Christopher, Curtis, Daniel, Eleanor, Erik, Fabiola, Fatemeh, Gabrielle, Jeremy, Jorge, Katayoun, Lora, Maria-Clara, Mason, Mathew, Mehwish, Mohammadmahdi, Nicholas, Oliver, Rachel, Rhupert, Samuel, Shivanshi, Utsav, William, Xuchen, Yihan.

Applied Communication Tasks

Applied Communication Tasks (ACTs) will give you the opportunity to develop and apply your knowledge of calculus in other situations, and learn additional quantitative material on your own.

There will be three Applied Communication Tasks introduced in tutorials throughout the term. You will work on them both in tutorials and at home, and submit them in tutorial. Your grade in this component of the course will be calculated according to the number of learning objectives that you demonstrate through the completion of the tasks throughout the term. You can find the learning objectives in the course Quercus page.

ACTs will be due in tutorials, so the exact due date will depend on when you have your tutorial; the weeks are shown below.

Tentative dates for ACTs:

	Draft	Peer Review	Final
ACT A	Before Tutorial 3	During Tutorial 3	Tutorial 4
ACT B	Before Tutorial 7	During Tutorial 7	Tutorial 8
ACT C	Before Tutorial 11	During Tutorial 11	December 8

Assessment

The following assessments are designed for an in-person class. If the class has to be moved online, the assessment structure and the grading scheme will change. There will be an announcement on Quercus in that case.

Start Here

This is an assignment meant for you to be acquainted with the different tools that we'll be using throughout the course. It will be available for you to complete in the first two weeks of classes.

Perusall and WeBWorK

Perusall is a platform where you can access the textbook and annotate it. Students will be divided in small groups and you can see each other's annotations. You can also ask questions to the other students in your group and you can answer questions from others.
Each week there will be an assignment to help you prepare for lectures. You will be graded on your comments and interactions:

- You need to write at least 5 comments, and each comment is graded according to its quality
- You will get a pass/fail for each assignment
- Your Perusall grade will be calculated from the best 8 assignments

WeBWorK. There will be a homework assignment at the end of each week on WeBWorK.

- Your WeBWorK grade will be calculated from the best 9 assignments

Quizzes

There will be two quizzes throughout the term. These quizzes are mandatory and written in your tutorial. You must write the quiz in person in the tutorial you are enrolled.

You will need to bring a tablet or a laptop to tutorial since part of the quiz will be done online. During the quiz, you can use online calculators like Desmos or Geogebra, and you can access the textbook, but you cannot consult with anybody else or use any other internet resource.

The tentative quiz dates are:

Quiz 1. In tutorial #2 on September 30–October 1.

Quiz 2. In tutorial #9 on November 25–26.

Term Test and Final Exam

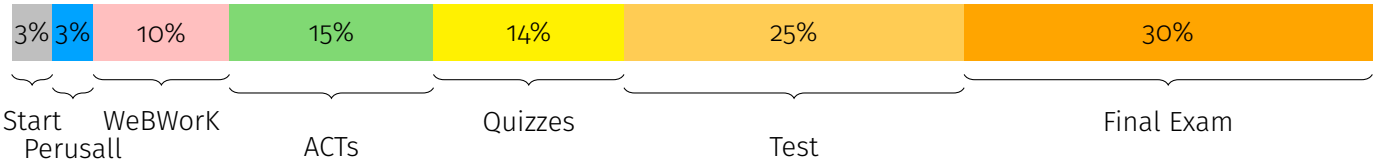
The Term Test and the Exam are common to all sections of MAT135 and will primarily consist of problems. Your solutions to these problems will be graded for both correctness and clarity. For many problems, it will not be enough to simply produce a correct final answer: you will need to show how you arrived at your answer by providing a complete solution. The questions will be based on the Learning Goals and Objectives given each week.

The tentative test date is:

Term test. on Friday, October 29 at 7–9pm.

Grading Scheme

In this course, you will be assessed based on your mastery of these learning objectives, not against other students in the class. Therefore your grades will not be 'curved' up or down. Since we are measuring your performance against these set criteria, we will not be releasing average grades or other information about how the class as a whole performs.



Email policy

- The University has a policy requiring that students have a U of T email address and that you check it regularly.
- If you email us from any address other than your @mail.utoronto.ca address, your email will be ignored. We would have no way to confirm your identity.
- If you have a question about the course policies, check the syllabus. Then check the FAQ on Quercus.
- If you send us an email, you are communicating with us in a professional context. Emails are not text messages or other social media interactions. Your email should start with a respectful greeting, have organized paragraphs and be signed with your name. We reserve the right to not consider an email that is written unprofessionally. All that said, don't worry too much about your grammar or spelling – we really do want to help you, however you write!
- Please do not email your TA unless they specifically asked you to do so.

Accessibility and Accommodations

The University provides academic accommodations for students with disabilities in accordance with the terms of the Ontario Human Rights Code.
If you have a learning need requiring an accommodation, please contact Accessibility Services as soon as possible at

https://www.studentlife.utoronto.ca/as

Academic Integrity

Familiarize yourself with the University of Toronto's Code of Behaviour on Academic Matters, available at

https://www.academicintegrity.utoronto.ca/

You are expected to know the rules. Keep in mind that not being aware of a rule is not an acceptable excuse for not having followed it. If you have any questions about what is or is not permitted in this course, please do not hesitate to talk to your instructor or TA.

Potential **offences** include, but are not limited to:

- Using someone else's ideas or words without appropriate acknowledgement
- Having anyone but the designated group member write a part of your tutorial worksheet
- Faking someone else's handwriting in a tutorial worksheet
- Using or possessing unauthorized aids during an exam, test, or quiz
- Looking at someone else's answers during an exam, test, or quiz
- Communicating with another student during an exam, test, or quiz
- Misrepresenting your identity
- Falsifying institutional documents or grades
- Falsifying or altering any documentation required by the University, including (but not limited to) forms related to a petition

Both receiving and providing unauthorized assistance is an academic offence. It does not matter if you "helped" or "were helped".
The following actions are **not offences** in this class:

- Discussing questions from homework with classmates, building off of each others' ideas
- Using online resources to help you understand the content of the course or homework problems
- Using online calculators (like desmos or geogebra) during a quiz

Copyright Notice

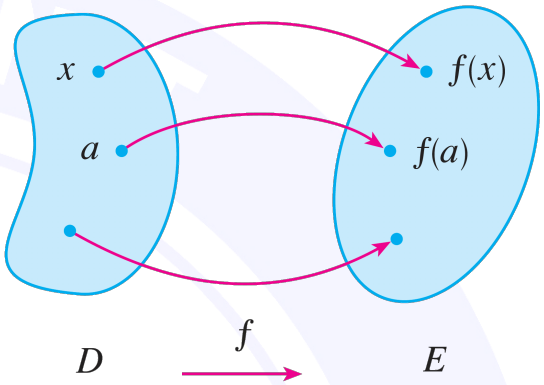
You must not take audio recordings or video recordings of lectures or tutorials unless you received the written consent of the person whose work you are recording (consent will normally be given for accessibility reasons).
There are on-campus and off-campus enterprises out there that offer so called "course preparation" and that will ask you to provide course material to them, sometimes in exchange for money, sometimes in exchange for "free help", sometimes without any direct benefit to you. You must not share the material of this course with such enterprises.

§1.1 Functions

Function

A **function** f is a rule that assigns to each element x in a set D exactly one element, called $f(x)$, in a set E.

- The set D is called the **domain** of the function. The number $f(x)$ is the **value of f at x** and is read "f of x".
- The **range** of f is the set of all possible value of $f(x)$ as x varies throughout the domain.
- A symbol that represents an arbitrary number in the ***domain*** of a function of f is called an **independent variable**.
- A symbol that represents a number in the ***range*** of f is called a **dependent variable**.

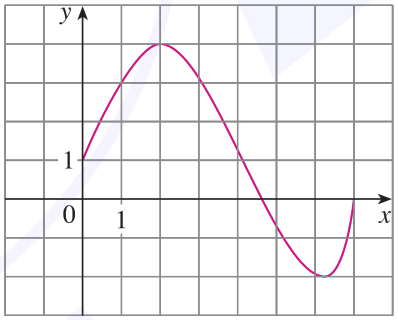


Exercise 1

The graph of a function f is shown on the right

(a) Find the values of $f(1)$ and $f(5)$.

(b) What are the domain and range of f?



Exercise 2

Sketch the graph and find the domain and range of each function.

(a) $f(x) = 2x - 1$

(b) $g(x) = x^2$

Exercise 3

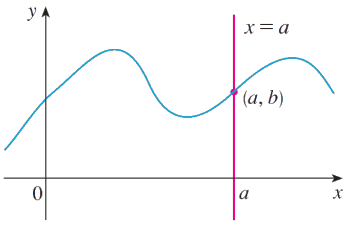
Find the domain of each function.

(a) $f(x) = \sqrt{x + 2}$

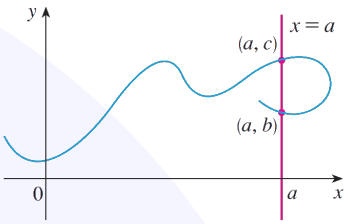
(b) $g(x) = \frac{1}{x^2 - x}$

The Vertical Line Test

A curve in the xy-plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.



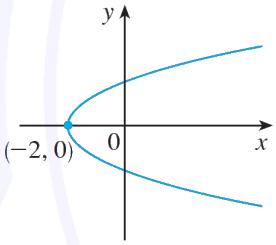
(a) This curve represents a function.



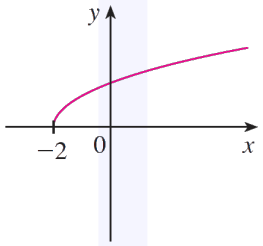
(b) This curve doesn't represent a function.

Example 1

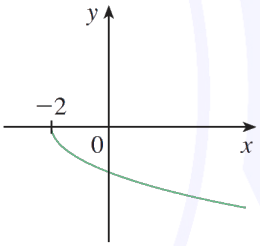
The parabola, $x = y^2 - 2$ is **not** the graph of a **function** of x because, as you can see, there are vertical lines that intersect the parabola twice. The parabola, however, does contain the graphs of two functions of x. Notice that the equation $x = y^2 - 2$ implies $y^2 = x + 2$, so $y = \pm\sqrt{x+2}$. Thus the upper and lower halves of the parabola are the graphs of the **functions** $f(x) = \sqrt{x+2}$ and $g(x) = -\sqrt{x+2}$.



(a) $x = y^2 - 2$



(b) $y = \sqrt{x+2}$



(c) $y = -\sqrt{x+2}$

Exercise 4

A **piecewise defined function** f is defined by

$$f(x) = \begin{cases} 1 - x & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$$

Evaluate $f(-2)$, $f(-1)$, and $f(0)$ and sketch the graph.

Symmetry

If a function f satisfies $f(-x) = f(x)$ for every number x in its domain, then f is called an **even function**.

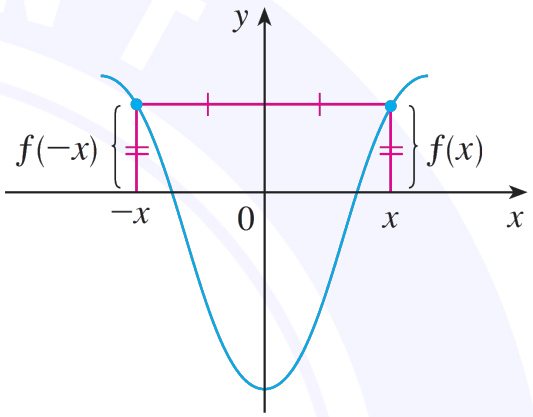
If f satisfies $f(-x) = -f(x)$ for every number x in its domain, then f is called an **odd function**.

Example 2

The function $f(x) = x^2$ is even because

$$f(-x) = (-x)^2 = x^2 = f(x)$$

The geometric significance of an even function is that its graph is symmetric with respect to the y-axis. This means if we have plotted the graph of f for $x \geq 0$, we obtain the entire graph simply by reflecting this portion about the y-axis.

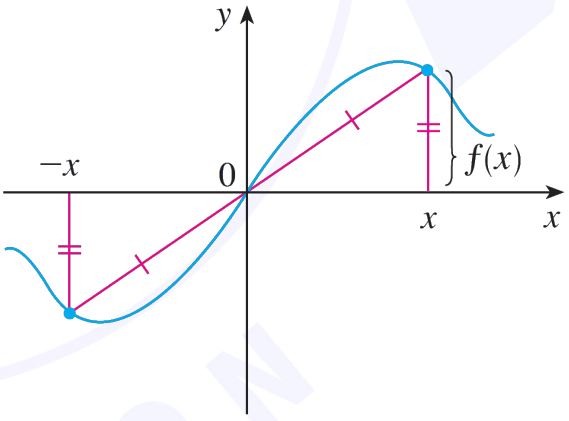


Example 3

The function $f(x) = x^3$ is odd because

$$f(-x) = (-x)^3 = -x^3 = -f(x)$$

The graph of an odd function is symmetric about the origin. If we already have the graph of f for $x \geq 0$, we can obtain the entire graph by rotating this portion about the origin.



Increasing and Decreasing Functions

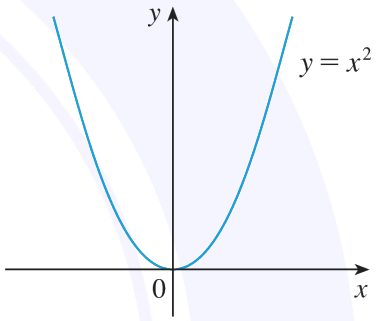
A function f is called **increasing** on an interval I if

$$f(x_1) < f(x_2) \text{ whenever } x_1 < x_2 \text{ in } I$$

It is called **decreasing** on I if

$$f(x_1) > f(x_2) \text{ whenever } x_1 < x_2 \text{ in } I$$

Example 4



In the definition of an increasing function it is important to realize that the inequality $f(x_1) < f(x_2)$ must be satisfied for every pair of number x_1 and x_2 in I with $x_1 < x_2$.

The function $f(x) = x^2$ is decreasing on the interval $(-\infty, 0]$ and increasing on the interval $[0, \infty)$.

§1.2 Mathematical Models

Linear Models

Y is a **linear function** of x means that the graph of the function is a line, so we can use the slope-intercept form of the equation of a linear to write a formula for the function as

$$y = f(x) = mx + b$$

where m is the slope of the line and b is the y-intercept.

Polynomials

A function P is called a **polynomial** if

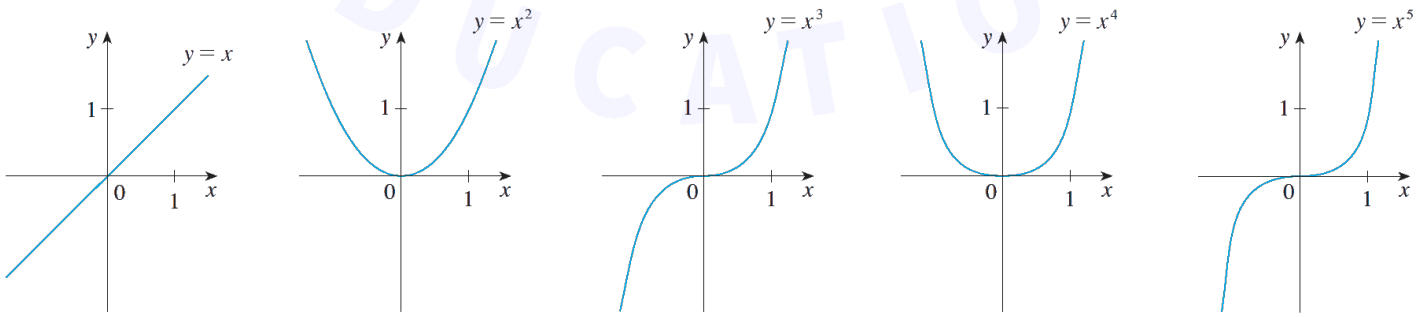
$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x^1 + a_0$$

where n is a nonnegative integer and the numbers $a_0, a_1, a_2, \ldots, a_n$ are constants called the **coefficients** of the polynomial. The domain of any polynomial is $\mathbb{R} = (-\infty, \infty)$. If the leading coefficient $a_n \neq 0$, then the **degree** of the polynomial is n.

Power Functions

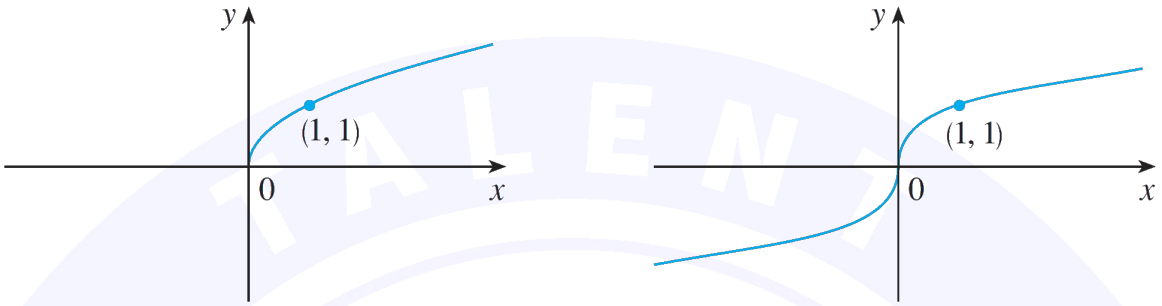
A function of the form $f(x) = x^a$, where a is a constant, is called a power function.

(i) $a = n$, where n is a positive integer



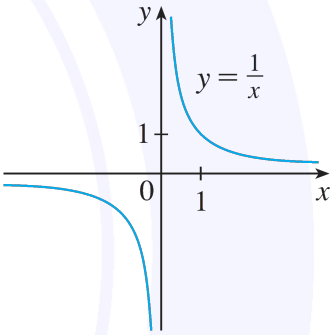
(ii) $a = \frac{1}{n}$, where n is a positive integer

The function $f(x) = x^{\frac{1}{n}} = \sqrt[n]{x}$ is a **root function**.



(a) $f(x) = \sqrt{x}$ (b) $f(x) = \sqrt[3]{x}$

(iii) $a = -1$



The graph of the **reciprocal function** $f(x) = x^{-1} = \frac{1}{x}$ is a hyperbola with the coordinate axes at its asymptotes.

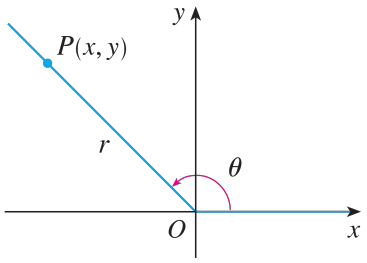
Rational Functions

A **rational function** f is a ratio of two polynomials:

$$f(x) = \frac{P(x)}{Q(x)}$$

where P and Q are polynomials. The domain of all values of x such that $Q(x) \neq 0$.

Trigonometric Functions



For a general angle θ in standard position we let $P(x, y)$ be any point on the terminal side of θ and we let r be the distance $|OP|$. Then we define

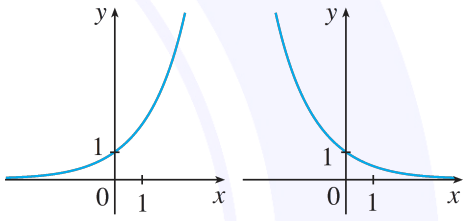
$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Exponential Functions

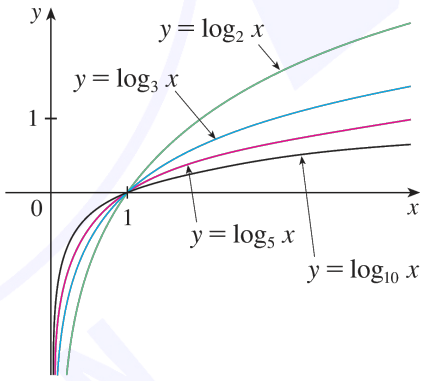
The **exponential functions** are the functions of the form $f(x) = b^x$, where the base b is a positive constant.



(a) $y = 2^x$ (b) $y = (0.5)^x$

Logarithmic Functions

The **logarithmic functions** $f(x) = \log_b x$, where the base b is a positive constant, are the inverse functions of the exponential functions.



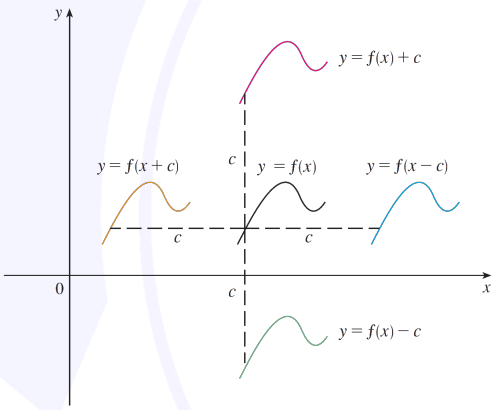
§1.3 New Functions From Old Functions

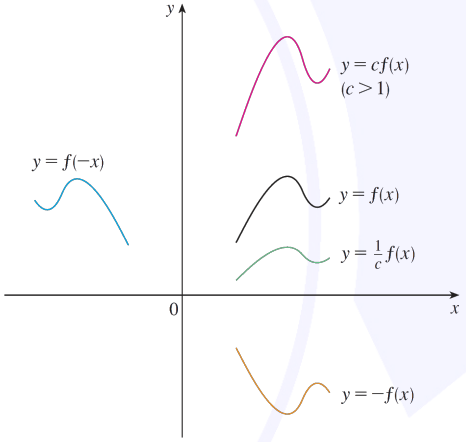
Transformations of Functions

Vertical and Horizontal Shifts (Translations)

Suppose $c > 0$. To obtain the graph of

- $y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward
- $y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward
- $y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right
- $y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left





Vertical and Horizontal Stretching and Reflecting

Suppose $c > 1$. To obtain the graph of

- $y = cf(x)$, stretch the graph of $y = f(x)$ vertically by a factor of c
- $y = \frac{f(x)}{c}$, shrink the graph of $y = f(x)$ vertically by a factor of c
- $y = f(cx)$, shrink the graph of $y = f(x)$ horizontally by a factor of c
- $y = f\left(\frac{x}{c}\right)$, stretch the graph of $y = f(x)$ horizontally by a factor of c
- $y = -f(x)$, reflect the graph of $y = f(x)$ about the x-axis
- $y = f(-x)$, reflect the graph of $y = f(x)$ about the y-axis

Exercise 1

Sketch the graphs of the following functions.

(a) $y = \sin 2x$

(b) $y = 1 - \sin x$

Exercise 2

Sketch the graph of the function $y = |x^2 - 1|$.

Combinations of Functions

Two functions f and g can be combined to form new function $f+g, f-g, fg$, and f/g in a manner similar to the way we add, subtract, multiply, and divide real numbers. The sum and difference functions are defined by

$$(f+g)(x) = f(x) + g(x) \qquad (f-g)(x) = f(x) - g(x)$$

Similarly, the product and quotient functions are defined by

$$(fg)(x) = f(x)g(x) \qquad \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Composition

Given two functions f and g, the **composite function** $f \circ g$ (also called the **composition** of f and g) is defined by

$$(f \circ g)(x) = f\big(g(x)\big)$$

Exercise 3

If $f(x) = x^2$ and $g(x) = x - 3$, find the composite functions $f \circ g$ and $g \circ f$.

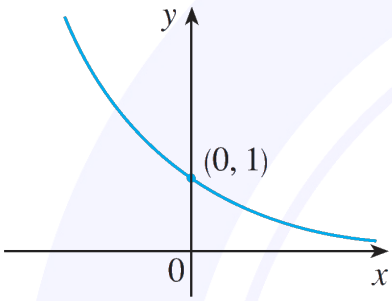
§1.4 Exponential Functions

Exponential Function

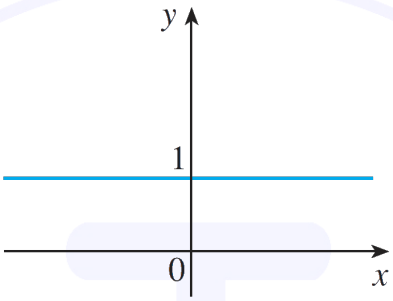
In general, an **exponential function** is a function of the form

$$f(x) = b^x$$

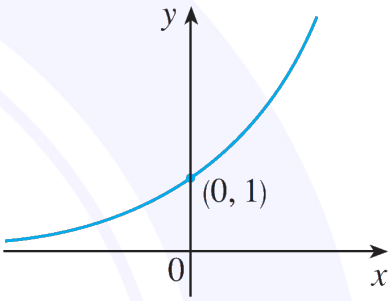
where b is a positive constant.



(a) $y = b^x,\ 0 < b < 1$



(b) $y = 1^x$



(c) $y = b^x,\ b > 1$

Law of Exponents

If a and b are positive numbers and x and y are any real numbers, then

1. $b^{x+y} = b^x b^y$
2. $b^{x-y} = \frac{b^x}{b^y}$
3. $(b^x)^y = b^{xy}$
4. $(ab)^x = a^x b^x$

Exercise 1

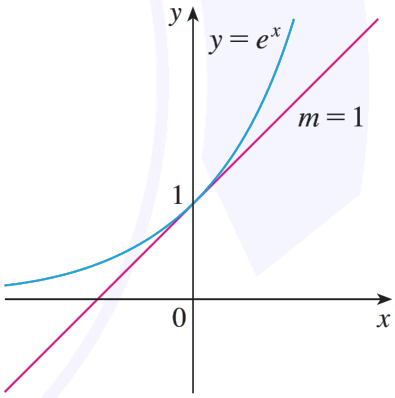
Sketch the graph of the function $y = 3 - 2^x$ and determine its domain and range.

Applications of Exponential Functions

The **half-life** is the time required for the viral load to be reduce to half its initial value.

The Number e

We call the function $f(x) = e^x$ the **natural exponential function**.

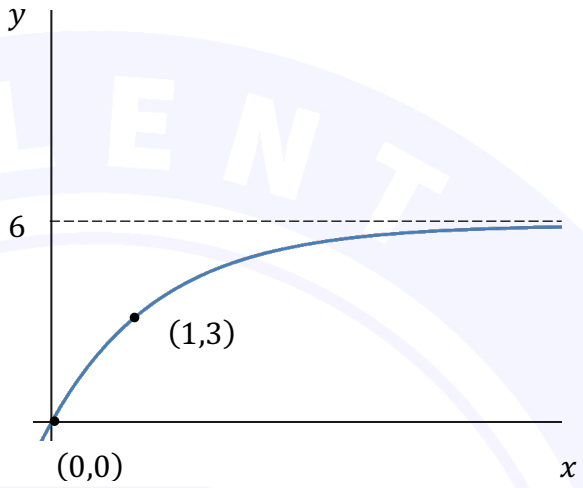


Exercise 2

Graph the function $y = \frac{1}{2}e^{-x} - 1$ and state the domain and range.

§1.1-1.4 Quiz

1. Give a possible formula for the graph



$y =$ ☐

2. In the early 1960s, radioactive strontium-90 was released during atmospheric testing of nuclear weapons and got into the bones of people alive at the time. If the half-life of strontium-90 is 29 years, what fraction of the strontium-90 absorbed in 1960 remained in people's bones in 2000?

[Hint: Write the function in the form $Q = Q_0\left(\frac{1}{2}\right)^{\frac{t}{29}}$.]

Round your answer to one decimal place.

☐ %

3. Is the function even, odd, or neither?

$$f(x) = e^{x^2-3}$$

- Even
- Odd
- Neither

4. If $f(x) = x^2 + 4$, simplify fully:

(a) $f(t+1) =$ ☐

(b) $f(t^2+1) =$ ☐

(c) $f(2) =$ ☐

(d) $\big(f(t)\big)^2 + 1 =$ ☐